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Exactly Solvable Model of Superstring in Plane-wave Background with Linear Null Dilaton

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Abstract

In this paper, we study an exactly solvable model of IIB superstring in a time-dependent plane-wave backgound with a constant self-dual Ramond-Ramond 5-form field strength and a linear dilaton in the light-like direction. This background keeps sixteen supersymmetries. In the light-cone gauge, the action is described by the two-dimensional free bosons and fermions with time-dependent masses. The model could be canonically quantized and its Hamiltonian is time-dependent with vanishing zero-point energy. The spectrum of the excitations is symmetric between the bosonic and fermionic sector. The string mode creation turns out to be very small.

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1 Introduction

The study of string theory in the time-dependent backgrounds has attracted much attention in the past decade. Similar to quantum field theory in the curved spacetime, string theory in the time-dependent backgrounds shares the same unclear subtle issues such as the choice of vacuum, string creation et al. Not mentioning a second-quantized version, even a perturbative description of string theory in a time-dependent background is in general out of control. Nevertheless, the issue is essential for us to understand the fundamental problems in quantum gravity. One central problem in quantum gravity is the resolution of the cosmic singularity. People have been expecting that the big-bang singularity could be resolved in string theory, namely the cosmological singularity should not exist in string theory. Unlike the usual orbifold or conifold singularity, the big-bang singularity is space-like and its resolution requires the knowledge of string theory in a time-dependent background.

In the past few years, there appeared a few remarkable models to address the issue. One class of such models is the time-dependent orbifolds [1, 2]. These models are solvable and keep part of supersymmetries. Unfortunately it turns out that the null orbifold background is unstable due to the large blueshift effect [3, 4] and the perturbative string theory in many time-dependent orbifolds breaks down [5, 6]. The α' correction and g_s correction play important roles near the singularity. Nevertheless, the recent development shows that the twisted states condensation may induce an RG-flow from null orbifold to the usual spatial orbifold [7]. Another class of solvable time-dependent models is the rolling tachyon, which shows the features such as tachyon condensation, unusual open/close string duality [8]. Very recently, several new ideas have been proposed. In [9], the authors argued that there is a closed string tachyon condensation phase replacing the cosmological singularity. In this case, the perturbative string amplitudes are self-consistently truncated to the small coupling region such that they are finite, indicating a resolution of the singularity.

Another interesting idea is so called matrix big bang. In [10], it has been proposed that in a linear null dilaton background it is the matrix degrees of freedom rather than the point particles or the perturbative strings that describe the physics near the big-bang singularity correctly. Some related discussions can be found in [11, 12, 13, 14], see also [15, 16]. The linear null dilaton background is different from the usual linear (spatial) dilaton. One obvious difference is that the latter one always leads to a noncritical string theory, while the former one does not modify the central charge or the dimension of the original theory. The common feature of two backgrounds is that due to the linearity of the dilaton, the

string coupling becomes very strong in some region where the perturbative string theory is not valid. In the linear spatial dilaton case, one can add a tachyon background (Liouville wall) to suppress the contribution coming from the strong coupling region in the path integral such that the perturbative theory is well-defined[17]. Due to the existence of the Liouville wall, the incoming wave is reflected and becomes outgoing. Among the theories with Liouville field, the 2D noncritical string theory with a time direction and a Liouville direction is of particular interest (See nice reviews [18]). It has a dual description by matrix model, which is exactly solvable. Actually, matrix model describes the dynamics of the D0-particles in 2D string theory[19]. This impressive relation between 2D string theory and matrix model is an illuminating manifestation of the open/closed duality. The story is a little different in the linear null dilaton case. It seems that one does not have a tachyon field to make the perturbative string theory well-defined. Instead, with the similar spirit leading to BFSS matrix model[38], a matrix model has been proposed to describe the physics in the strong coupling region[10]. Similarly, the matrix degrees of freedom are identified with the open string degrees of freedoms between the partons.

Inspired by the recent interest in the linear null dilaton background, we will study an exactly solvable string model in a plane-wave background in the presence of self-dual Ramond-Ramond(RR) 5-form field strength and a linear dilaton in the light-cone direction. The study of string theory in a plane-wave background has a long history. It is remarkable that [20] the plane-fronted waves are not only the solutions of the effective supergravity but even the exact solutions of the string theory. Furthermore in [21, 22] the IIB superstring in the plane-wave background with a constant self-dual Ramond-Ramond field strength and a constant dilaton is quantized in the lightcone gauge. This leads to the famous PP-wave/SYM correspondence[24]. Later on, a time-dependent plane-wave string model was studied in [25].

In this paper we consider the type IIB Green-Schwarz superstring in the following time-dependent background with Ramond-Ramond flux:

$$
\begin{aligned}
&ds^2 = -2dx^+dx^- - \lambda(x^+)\, x_I^2\, dx^+dx^+ + dx^I dx^I\,, \\
&\phi = \phi(x^+)\,, \qquad (F_5)_{+1234} = (F_5)_{+5678} = 2f.
\end{aligned} \tag{1.1}
$$

The world-sheet conformal invariance requires

$$
R_{\mu\nu} = -2D_\mu D_\nu \phi + \frac{1}{24} e^{2\phi} (F_5^2)_{\mu\nu}. \tag{1.2}
$$

The only nonzero component of the Ricci curvature tensor $R_{\mu\nu}$ with respect to the metric in (1.1) is $R_{++} = 8\lambda(x^+)$. Inserting (1.1) into the conformal invariance condition (1.2),

we get the relation

$$\lambda = -\frac{1}{4}\phi'' + f^2 e^{2\phi} \,. \tag{1.3}$$

From now on we restrict the dilaton to be linear in the light-cone time coordinate x^+, i.e. $\phi = -cx^+$ with c being a constant, then we have

$$\lambda = f^2 e^{-2cx^+}. \tag{1.4}$$

In general, f could be an arbitrary function of x^+ and there is a large class of the models. These models could be studied in the lightcone gauge. For a generic f, it is difficult to solve the model analytically. However, there are two special cases:

- One is that λ is a constant. This happens when $f = f_0 e^{-\phi}$ with f_0 being constant so that

  $$\lambda = f_0^2 \,, \tag{1.5}$$

  and the metric in the string frame reduces to the form of the maximally supersymmetric plane wave [23]. Actually string theory in this background is very similar to the one in the maximally supersymmetric case. The bosonic and fermionic action, the quantization and the spectrum are all the same. However, the vertex operators and the perturbative amplitudes are different due to the existence of the linear dilaton. It could be expected that the perturbative string theory is not well-defined in the strong coupling region and an alternative description, say the matrix degrees of freedom, is needed.

- The other special case is when the self-dual RR field strength $f = f_0$ is constant. This is the case we will pay most of our attention to in this paper. In this case, the perturbative string theory is still exactly solvable. We manage to quantize both the bosonic and fermionic sectors of string theory in the light-cone gauge. It turns out that the Hamiltonian is time-dependent and the zero-point energy cancels between the bosonic and fermionic sectors. And the spectrum of the bosonic and fermionic excitations is symmetric. Similarly the string coupling is very strong near the big bang singularity.

Note that without loss of generality the coefficient c in the dilaton can be set to any value by rescaling the coordinates x^+, x^- and the RR field strength f properly. In the following sections when we solve this model in the light-cone gauge, we will set $c = \frac{1}{\alpha' p^+}$ for simplicity. In this case, c is positive and indicates that the strong string coupling region is near $x^+ \to -\infty$.

The paper is organized as follows. In section 2, we investigate some properties of the background: its symmetry algebra, the geodesic incompleteness, supersymmetries and its description in Rosen coordinates. In section 3, we study the quantization of the bosonic sector. In section 4, we turn to the fermionic sector, which needs more techniques. In section 5, we discuss the quantum string mode creation. We end the paper with the conclusion and some discussions.

2 Some Properties of the Background

Let us start from the metric in (1.1) and discuss some properties of the background.

2.1 Symmetry Algebra of the background

We are interested in the symmetry algebra of the background, which is encoded in the complete set of Killing vectors preserving the background (1.1). It is manifest that the background is invariant under translation in the x^- direction, and the corresponding Killing vector is

$$T \equiv \frac{\partial}{\partial x^-} , \tag{2.1}$$

which generates a $\mathbb{R}$-subgroup of the total isometry group. But unlike the maximally symmetric case in [23], the translation invariance in the x^+ direction is broken by the nontrivially x^+-dependent dilaton. Although the metric does not possess $SO(8)$ symmetry due to the presence of RR five-form flux, the background is invariant under two $SO(4)$'s, which act on x^i and x^a directions separately. We denote the corresponding generators by J_{ij} and J_{ab} with i, j running from 1 to 4 and a, b from 5 to 8,

$$J_{ij} \equiv x_i \partial_j - x_j \partial_i \quad , \quad J_{ab} \equiv x_a \partial_b - x_b \partial_a \ . \tag{2.2}$$

If we change these two $SO(4)$'s, the solution is still invariant. So we get a $\mathbb{Z}_2$ symmetry,

$$\{x^i\} \longleftrightarrow \{x^a\}. \tag{2.3}$$

It is evident that the translations along $x^I = \{x^i, x^a\}$ do not leave the solution invariant. However, if we shift x^- appropriately at the same time, we find new symmetric translations with the generators

$$L_I = a\partial_I + (\partial_+ a)\, x_I \partial_- \, , \tag{2.4}$$

with $a(x^+)$ satisfying

$$\partial_+^2 a + \lambda\, a = 0\,. \tag{2.5}$$

Since (2.5) is a 2-order equation, we get two sets of generators denoted by L_I, $\tilde{L}_I$.

1. In the case $\lambda = f_0^2$, we have

$$\begin{aligned} L_I &= \cos(f_0 x^+)\partial_I - f_0 \sin(f_0 x^+) x_I \partial_-\,, \\ \tilde{L}_I &= \sin(f_0 x^+)\partial_I + f_0 \cos(f_0 x^+) x_I \partial_-\,. \end{aligned} \tag{2.6}$$

2. In the case $\lambda = f_0^2\, e^{-2cx^+}$, we get

$$\begin{aligned} L_I &= J_0\left(\frac{f_0}{c}\, e^{-cx^+}\right)\partial_I + f_0\, e^{-cx^+} J_1\left(\frac{f_0}{c}\, e^{-cx^+}\right) x_I \partial_-\,, \\ \tilde{L}_I &= Y_0\left(\frac{f_0}{c}\, e^{-cx^+}\right)\partial_I + f_0\, e^{-cx^+} Y_1\left(\frac{f_0}{c}\, e^{-cx^+}\right) x_I \partial_-\,. \end{aligned} \tag{2.7}$$

It is straightforward to write down the non-vanishing commutators as follows :

$$\begin{aligned} &[L_I,\ \tilde{L}_J] = \gamma T \delta_{IJ}\,, \\ &[J_{IJ},\, L_K] = L_I \delta_{JK} - L_J \delta_{IK}\,, \\ &[J_{IJ},\, \tilde{L}_K] = \tilde{L}_I \delta_{JK} - \tilde{L}_J \delta_{IK}\,, \\ &[J_{IJ}, J_{KL}] = J_{IL}\delta_{JK} + J_{JK}\delta_{IL} - J_{IK}\delta_{JL} - J_{JK}\delta_{IL}\,, \end{aligned} \tag{2.8}$$

where

$$\gamma = \begin{cases} f_0, & \text{when } \lambda = f_0^2, \\ -\frac{2c}{\pi}, & \text{when } \lambda = f_0^2\, e^{-2cx^+}. \end{cases} \tag{2.9}$$

Here we have used the identity $J_0(z)Y_1(z) - J_1(z)Y_0(z) = -\frac{2}{\pi z}$ to get the first commutator in the second case and for simplicity we have defined $J_{IJ} = \{J_{ij}, J_{ab}\}$. From the commutators above, we can see L_I, $\tilde{L}_I$ and T form the Heisenberg-type algebra $h(8)$ with $\hbar$ being replaced by γ. Therefore in both cases the continuous symmetry algebra is $[so(4) \oplus so(4)] \oplus_s h(8)$. Here $\oplus_s$ means semi-direct sum. So the background (1.1) admits a symmetry algebra of dimension twenty-nine.

Another way to check the symmetry algebra is to work in the Einstein frame and figure out the Killing vectors which keep the RR background invariant. Following the discussion in [12], it is straightforward to find that the Killing vector corresponding to the translational invariance along x^+ does not exist, which indicates the absence of the supernumerary supersymmetries. In fact, the Killing symmetries discussed above are the

complete set preserving the background. It is not hard to check that our background is inhomogeneous.

With the above Killing vectors, there are a few physical implications. Firstly one may define the conserved quantities with the Killing vectors. For instance, the conserved quantity corresponding to the translational invariance Killing vector T is the lightcone momentum. On the other hand, the absence of the Killing vector ∂_+ indicates that the lightcone Hamiltonian is not conserved. Actually we will show that the Hamiltonian in our background is time-dependent. Another implication is that due to the existence of the null Killing vector T, there is no particle or string creation in our background [30].

2.2 Geodesics and Tidal force

In this subsection we discuss some geometric features of our background according to the general method of [26]. Using the metric in our background (1.1), we can obtain the non-vanishing components of the Christoffel connection

$$\Gamma^{-}_{++} = -c\lambda x_I^2 \quad , \quad \Gamma^{-}_{+I} = \Gamma^{I}_{++} = \lambda x_I \quad . \tag{2.10}$$

It is easy to get the nonzero components of the Riemann tensor

$$R_{I+I+} = \lambda \ . \tag{2.11}$$

In our background the geodesic equation

$$\frac{d^2 x^\mu}{d\sigma^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\sigma}\frac{dx^\beta}{d\sigma} = 0 \tag{2.12}$$

can be rewritten as

$$\begin{aligned} &\frac{d^2x^+}{d\sigma^2} = 0 \ , \\ &\frac{d^2x^-}{d\sigma^2} + 2\lambda x_I \frac{dx^+}{d\sigma}\frac{dx^I}{d\sigma} - c\lambda x_I^2 \frac{dx^+}{d\sigma}\frac{dx^+}{d\sigma} = 0 \ , \\ &\frac{d^2x^I}{d\sigma^2} + \lambda x_I \frac{dx^+}{d\sigma}\frac{dx^+}{d\sigma} = 0 \ , \end{aligned} \tag{2.13}$$

where σ is an affine parameter.

The general solution of the first equation in (2.13) is

$$x^+ = x_0^+\sigma + x_1^+ \ , \tag{2.14}$$

where x_0^+ and x_1^+ are constants. If $x_0^+ = 0$, we get

$$x^+ = x_1^+, \qquad x^- = x_0^-\sigma + x_1^-, \qquad x^I = x_0^I\sigma + x_1^I \ , \tag{2.15}$$

with x_0 and x_1 being constants. These geodesics are as in flat space. If $x_1^+ \neq 0$, for simplicity we write $x^+ = \sigma$, which is equivalent to rescale σ and shift its origin. It is easy to check that the curve

$$x^+ = \sigma, \qquad x^- = 0, \qquad x^I = 0 \ , \tag{2.16}$$

is a geodesic. Using the translation e^ξ to the curve (2.16), we can obtain a new family of null geodesics. Here ξ is the Killing field associated with (2.4) and $\xi^\mu = (0,\, x^I\partial_+ a,\, \xi^I)$ with $\xi^I = a$ defined in (2.5). These new geodesics are

$$x^+ = \sigma, \qquad x^- = \frac{1}{2}\xi^I\partial_+\xi_I, \qquad x^I = \xi^I \ . \tag{2.17}$$

The distance separating the geodesics (2.16) and (2.17) along any surface $x^+ = constant$ is given by $|\xi| = \sqrt{|\xi_I\xi^I|}$. Since x^+ is an affine parameter for this family of geodesics, it is meaningful to speak of the relative velocity $\partial_+\xi^I$ and acceleration $\partial_+^2\xi^I$ of these geodesics. Note that these relative accelerations measure gravitational tidal forces. If the geodesics are infinitesimally separated, the tidal force is given by certain components of the Riemann tensor:

$$\partial_+^2\xi^I = -R^I_{\alpha\beta\gamma}\xi^\beta t^\alpha t^\gamma \ , \tag{2.18}$$

where t^α is the tangent vector to the geodesic. In our case, (2.18) takes the form

$$\partial_+^2 a = -\lambda\, a \ , \tag{2.19}$$

which is just the equation (2.5) and λ characterizes the strength of the tidal force. We can see that as $x^+ \to -\infty$, the tidal force becomes divergent in the case with constant RR field strength. And since λ is positive, the tidal force is always attractive.

The above discussions are in the string frame. Due to the existence of the linear null dilaton, the test particle or string will interact with the dilaton, which drives the test particle away from the geodesics above. It is better to work in the Einstein frame. Things become more subtle then. In the Einstein frame, the metric in our background (1.1) can be written as

$$ds_E^2 = e^{-\frac{\phi}{2}} ds_{str.}^2 \ . \tag{2.20}$$

The non-vanishing components of the corresponding Christoffel connection are

$$\begin{aligned}
&\Gamma^+_{++} = \frac{c}{2} \ , \\
&\Gamma^-_{++} = -\frac{5}{4}c\lambda x_I^2 \quad , \quad \Gamma^-_{+I} = \lambda x_I \quad , \quad \Gamma^-_{II} = \frac{c}{4} \ , \\
&\Gamma^I_{++} = \lambda x_I \quad , \quad \Gamma^I_{+I} = \frac{c}{4} \ .
\end{aligned} \tag{2.21}$$

and the nonzero components of the Riemann tensor take the form

$$R_{I+I+} = \frac{c^2}{16} + \lambda \ . \tag{2.22}$$

It's easy to see that the Einstein metric has a singularity at $x^+ \to -\infty$ since some of metric components go to zero. Actually such a singularity occurs at the finite geodesic distance, which indicates the spacetime is geodesically incomplete. Let us focus on the lines $x^I = 0, x^- =$ constant, which are geodesics, and consider the geodesic equation for x^+

$$\frac{d^2x^+}{d\sigma^2} + \frac{c}{2}\left(\frac{dx^+}{d\sigma}\right)^2 = 0 \ , \tag{2.23}$$

which gives

$$e^{\frac{c}{2}x^+}(\frac{dx^+}{d\sigma}) = \text{constant.} \tag{2.24}$$

Hence the affine parameter is

$$\sigma = e^{\frac{c}{2}x^+} \tag{2.25}$$

up to an affine transformation. Therefore the singularity $x^+ \to -\infty$ corresponds to $\sigma = 0$ and it has finite affine distance to all points in the interior. In terms of the affine parameter σ, the metric could be rewritten as

$$ds_E^2 = -\frac{4}{c}d\sigma dx^- - \left(\frac{2f}{c}\right)^2 \sigma^{-5}x_I^2 d\sigma^2 + \sigma dx_I^2 \tag{2.26}$$

and the nonvanishing components of the corresponding Riemann tensor are

$$R_{\sigma I \sigma I}\frac{1}{4\sigma^2} + \left(\frac{2f}{c}\right)^2 \frac{1}{\sigma^6}, \tag{2.27}$$

which shows a curvature singularity at $\sigma = 0$ and gives a divergent tidal force felt by an inertial observer.

In short, our background (1.1) is geodesically incomplete and hence singular from the standpoint of general relativity. This background is analogue to those with a cosmological singularity. Near the singularity, the test particle experiences divergent gravitational tidal force, while for the string the string interaction becomes extremely strong and the string feels divergent tidal force. One needs a nonperturbative description of the string theory near the cosmological singularity.

2.3 Supersymmetries of the background

Now let's check that half of the maximally space-time supersymmetry has been broken in our background. To this end, we need to know how many independent Killing spinors the background (1.1) supports. The gravitino and dilatino variations should vanish for independent Killing spinors, i.e.

$$\delta_\epsilon \lambda^A \equiv (\tilde{\mathcal{D}})^A_B\, \epsilon^B = 0\,, \quad \delta_\epsilon \psi^A_\mu \equiv (\hat{\mathcal{D}}_\mu)^A_B\, \epsilon^B = 0\,, \tag{2.28}$$

where $\mu = +, -, 1, ..., 8$ and $A = 1, 2$. The dilatinos λ^A, gravitinos ψ^A_μ, and Killing spinors ϵ^A are all ten dimensional Weyl-Majorana fermions of positive chirality, so they carry a 10d spinor index, $\alpha = 1, 2, \ldots, 32$, additionally, e.g. ϵ^A_α etc. (for our notations and conventions about spinors see Appendix A). In the string frame, the generalized covariant derivative $\tilde{\mathcal{D}}$ and $\hat{\mathcal{D}}_\mu$ for our background (1.1) are [27, 28]

$$(\tilde{\mathcal{D}})^A_B = \frac{1}{2}\,\delta^A_B\,\Gamma^\mu\,\partial_\mu\phi\,, \tag{2.29}$$

$$(\hat{\mathcal{D}}_\mu)^A_B = \delta^A_B\,\partial_\mu + (\Omega_\mu)^A_B\,, \tag{2.30}$$

with

$$(\Omega_\mu)^A_B = \frac{1}{4}\omega_\mu^{\hat{\nu}\hat{\rho}}\Gamma_{\hat{\nu}\hat{\rho}}\,\delta^A_B + \frac{i\,e^\phi}{8\cdot 5!}\,\Gamma^{\kappa\nu\rho\sigma\delta}F_{\kappa\nu\rho\sigma\delta}\Gamma_\mu(\sigma_2)^A_B\,, \tag{2.31}$$

where σ_2 is the Pauli matrix, $\omega_\mu^{\hat{\nu}\hat{\rho}}$ is the spin connection and the hatted indices are used for the tangent space. Since $\phi = \phi(x^+)$, the dilatino variation imposes the constraint

$$\Gamma^+\epsilon^A = 0, \quad A = 1, 2\,. \tag{2.32}$$

Therefore at most the background (1.1) admits 16 supersymmetries. Now let's consider the gravitino variations. In order to work out the spin connection $\omega_\mu^{\hat{\nu}\hat{\rho}}$, we choose the vierbeins $e_\mu^{\hat{\nu}}$ as follows

$$e_+^{\hat{+}} = e_-^{\hat{-}} = 1\ ,\ e_J^{\hat{I}} = \delta_J^{\hat{I}}\ ,\ e_+^{\hat{-}} = \frac{1}{2}\lambda\, x_I^2\ , \tag{2.33}$$

It is easy to get the only non-vanishing components of $\omega_\mu^{\hat{\nu}\hat{\rho}}$ are

$$\omega_+^{\hat{-}\hat{I}} = \lambda\, x^I\,. \tag{2.34}$$

In general the Gamma matrices with respect to the coordinate, say Γ^μ, Γ_μ, and those with respect to the vierbein, say $\Gamma^{\hat{\mu}}, \Gamma_{\hat{\mu}}$, are not the same things. They are related by $\Gamma^\mu = \Gamma^{\hat{\mu}} e^\mu_{\hat{\mu}}$ and $\Gamma_\mu = \Gamma_{\hat{\mu}} e^{\hat{\mu}}_\mu$. In our background we have

$$\begin{aligned} &\Gamma^I = \Gamma^{\hat{I}}\,, \quad \Gamma^+ = \Gamma^{\hat{+}}\,, \quad \Gamma_+ = \Gamma_{\hat{+}} + \frac{1}{2}\lambda\, x_I^2\,\Gamma_{\hat{-}}\,, \\ &\Gamma_I = \Gamma_{\hat{I}}\,, \quad \Gamma_- = \Gamma_{\hat{-}}\,, \quad \Gamma^- = \Gamma^{\hat{-}} - \frac{1}{2}\lambda\, x_I^2\,\Gamma^{\hat{+}}\,. \end{aligned} \tag{2.35}$$

Now we will consider the Killing spinor equation coming from gravitino variation

$$(\mathbf{1}\cdot\partial_\mu+\Omega_\mu)^A_B\,\epsilon^B=0\ , \tag{2.36}$$

with

$$\begin{aligned}\Omega_- &= 0\ ,\\ \Omega_I &= \frac{i\,e^\phi}{4}f\,\Gamma^+(\Pi+\Pi')\,\Gamma_I\,\sigma_2\ ,\\ \Omega_+ &= -\frac{1}{2}\lambda\,x^I\Gamma^{+I}\,\mathbf{1}+\frac{i\,e^\phi}{4}f\,\Gamma^+(\Pi+\Pi')\,\Gamma_+\sigma_2\ ,\end{aligned} \tag{2.37}$$

where $\Pi=\Gamma^{1234}=\Gamma^1\Gamma^2\Gamma^3\Gamma^4$ and $\Pi'=\Gamma^{5678}=\Gamma^5\Gamma^6\Gamma^7\Gamma^8$. Taking (2.32) and $(\Gamma^+)^2=0$ into account, we note that the $\mu=-,I$ components of (2.36) are simply

$$\partial_-\,\epsilon^A=0\,,\quad \partial_I\,\epsilon^A=0\,. \tag{2.38}$$

Therefore ϵ^A is x^- and x^I-independent. The $\mu=+$ component of the Killing spinor equation (2.36) takes the form

$$(\mathbf{1}\cdot\partial_+ +i\,e^\phi f\,\Pi\,\sigma_2)^A_B\ \epsilon^B=0\ , \tag{2.39}$$

where we have used the facts $\Pi\epsilon^A=\Pi'\epsilon^A$ and $\Gamma^+\Gamma_+\epsilon^A=2\epsilon^A$ due to $\Gamma^+\epsilon^A=0$. The equation (2.39) can be written as

$$\begin{aligned}&\partial_+\epsilon^1+e^\phi f\,\Pi\,\epsilon^2=0,\\ &\partial_+\epsilon^2-e^\phi f\,\Pi\,\epsilon^1=0\ .\end{aligned} \tag{2.40}$$

1. In the case of $f=f_0\,e^{-\phi}$, the equation (2.40) becomes

$$\begin{aligned}&\partial_+\epsilon^1+f_0\,\Pi\,\epsilon^2=0,\\ &\partial_+\epsilon^2-f_0\,\Pi\,\epsilon^1=0\ .\end{aligned} \tag{2.41}$$

   The corresponding solutions are

$$\begin{aligned}\epsilon^1&=\chi_0\cos(f_0\,x^+)+\Pi\tilde\chi_0\sin(f_0\,x^+)\,,\\ \epsilon^2&=-\tilde\chi_0\cos(f_0\,x^+)+\Pi\chi_0\sin(f_0\,x^+)\,;\end{aligned} \tag{2.42}$$

2. In the case of $f=f_0$, the solutions of equation (2.40) are given by

$$\begin{aligned}\epsilon^1&=\chi_0\cos\frac{f_0\,e^{-cx^+}}{c}+\Pi\tilde\chi_0\sin\frac{f_0\,e^{-cx^+}}{c}\ ,\\ \epsilon^2&=\tilde\chi_0\cos\frac{f_0\,e^{-cx^+}}{c}-\Pi\chi_0\sin\frac{f_0\,e^{-cx^+}}{c}\ .\end{aligned} \tag{2.43}$$

Here χ_0 and $\tilde{\chi}_0$ are arbitrary constant 10d Majorana-Weyl spinors of positive chirality and satisfy (2.32). Therefore the background (1.1) preserves 16 supersymmetries. In fact, the above discussion does not depend on the explicit form of the λ and f, so any supergravity solutions satisfying (1.4) keep half of the original supersymmetries.

2.4 Asymptotic behavior in Rosen coordinates

The background (1.1) with linear null dilaton and constant RR five-form flux seems to support very smooth geometry. Using the metric in (1.1), we get the only non-vanishing component of the Ricci tensor is $R_{++} = 8\lambda$. In the case with constant RR field strength, $\lambda = f_0^2\, e^{-2c\,x^+}$. When x^+ goes from $-\infty$ to $+\infty$, λ decreases from $+\infty$ to zero . Taking the x^+ as light-cone time, the evolution is to start from the *big bang* region in the past and reach the flat space in the infinite future. In the other case with constant λ, the evolution is from a *big bang* singular point to the plane-wave spacetime in the light-cone future. Similar to the case studied in [10], we have a big-bang singularity in the Einstein frame.

To see it clearly, it is better to transform to Rosen coordinates. ¿From Brinkmann coordinates $(x^+,\, x^-,\, x^I)$ to Rosen coordinates $(x^+,\, \tilde{x}^-,\, \tilde{x}^I)$, we change the coordinates in the following way

$$x^- = \tilde{x}^- + \frac{1}{2}\, a a' \tilde{x}^I \tilde{x}^I \ ,\ x^I = a\tilde{x}^I \ , \tag{2.44}$$

with a is defined in (2.5). Now the metric takes the form

$$ds^2 = -2dx^+ d\tilde{x}^- + a^2(x^+) d\tilde{x}^I d\tilde{x}^I \ , \tag{2.45}$$

and thus is conformally flat. In the case with constant RR field strength, the solution of the equation (2.5) is

$$a(x^+) = C_1\, J_0\left(\frac{f_0}{c}\, e^{-cx^+}\right) + C_2\, Y_0\left(\frac{f_0}{c}\, e^{-cx^+}\right) \ , \tag{2.46}$$

where J and Y are standard Bessel and Neumann functions ; C_1 and C_2 are constants. The diverse values of (C_1, C_2) can be thought of different conformal embeddings of the plane wave into Minkowski spacetime. Since J_0 and Y_0 are both oscillating functions, a oscillates with time.

Using the asymptotic expansion of Bessel functions as $z \to +\infty$,

$$\begin{aligned} J_\nu(z) &\sim \sqrt{\frac{2}{\pi z}}\cos(z - \frac{\nu\pi}{2} - \frac{\pi}{4}) \ , \quad |\arg z| < \pi \ , \\ Y_\nu(z) &\sim \sqrt{\frac{2}{\pi z}}\sin(z - \frac{\nu\pi}{2} - \frac{\pi}{4}) \ , \quad |\arg z| < \pi \ , \end{aligned} \tag{2.47}$$

we can see that as $x^+ \to -\infty$,$\frac{f_0}{c}\, e^{-cx^+} \;\to \infty$,

$$\begin{aligned}
&J_0 \sim e^{\frac{c}{2}x^+}\sqrt{\frac{2\,c}{\pi f_0}}\,\cos\left(\frac{f_0}{c}\,e^{-cx^+}\right) \to 0\ , \\
&Y_0 \sim e^{\frac{c}{2}x^+}\sqrt{\frac{2\,c}{\pi f_0}}\,\sin\left(\frac{f_0}{c}\,e^{-cx^+}\right) \to 0\ , \\
&a(x^+) \to 0\ ,
\end{aligned} \tag{2.48}$$

which corresponds to the big bang singularity in the Brinkmann coordinates. Considering the asymptotic expansion of Bessel functions as $z \to 0$,

$$\begin{aligned}
J_\nu(z) &\sim \frac{1}{\Gamma(1+\nu)}\left(\frac{z}{2}\right)^\nu + \mathcal{O}(z^{\nu+2})\ , \\
Y_0(z) &\sim \frac{2}{\pi}\ln\frac{z}{2}\ ,
\end{aligned} \tag{2.49}$$

we get as $x^+ \to +\infty$, $\frac{f_0}{c}\, e^{-cx^+}\!\to 0$,

$$\begin{aligned}
&J_0 \sim 1\ \ ,\ Y_0 \sim \frac{2}{\pi}(\ln\frac{f_0}{c} - cx^+)\ , \\
&a(x^+) \sim C_1 + C_2\left(\ln\frac{f_0}{c} - cx^+\right).
\end{aligned} \tag{2.50}$$

In order to recover the flat space limit, we set $C_1 = 1$ and $C_2 = 0$. Thus

$$a(x^+) = J_0\left(\frac{f_0}{c}\,e^{-cx^+}\right)\ . \tag{2.51}$$

Therefore, in the Rosen coordinate, the metric in the string frame looks like the Friedman-Robersen-Walker metric but now the scale factor oscillates with the light-cone time.

3 Bosonic sector

It is well-known that the plane-wave backgrounds are the exact solutions to the string theory if they solve the supergravity equations [31, 32, 20]. In other words, the plane-wave background is exact against the α' correction. And in [20] it has been shown that the light-cone gauge can be implemented for the string theory in a plane-wave background. This has been generalized to the string theory in a plane-wave background with RR field strength in [21, 22]. In this and the next sections we will solve the first-quantized type IIB free superstring model in the background (1.1) with linear null dilaton and constant RR five-form flux. In the light-cone gauge the Green-Schwarz action is quadratic in the

transverse bosonic and fermionic coordinates for the plane wave metric with any function $\lambda(x^+)$, so we can explicitly write down the classical solutions, perform the canonical quantization and find the the light-cone Hamiltonian in terms of creation and annihilation operators.

First we will study the bosonic sector of the model. The bosonic part of the GS action in the background (1.1) is

$$\begin{aligned} S_B &= -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-g}\, g^{ab}G_{\mu\nu}\partial_a x^\mu \partial_b x^\nu \\ &= -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-g}\, g^{ab}(-2\,\partial_a x^+\partial_b x^- - \lambda\, x_I^2\partial_a x^+\partial_b x^+ + \partial_a x^I\partial_b x^I)\,. \end{aligned} \tag{3.1}$$

First we choose the world-sheet metric as

$$\sqrt{-g}\, g^{ab} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{3.2}$$

To fix the residual worldsheet gauge symmetries, we note the equation of motion for x^+ is $(\partial_\tau^2 - \partial_\sigma^2)x^+ = 0$, which has a general solution of the form $f(\tau+\sigma) + g(\tau-\sigma)$. We choose $f(x) = g(x) = \frac{1}{2}\alpha' p^+ x$, then

$$x^+ = \alpha' p^+ \tau \ ,\ p^+ > 0\,. \tag{3.3}$$

The conditions (3.2) and (3.3) completely fix the world-sheet gauge symmetries of the bosonic action (3.1). This is the well-known light-cone gauge. After imposing these gauge choices, x^- is not a dynamical variable either, which is completely determined by x^I through the constraints resulting from

$$\frac{\delta S_B}{\delta g^{\tau\sigma}} = 0\ ,\ \frac{\delta S_B}{\delta g^{\tau\tau}} = \frac{\delta S_B}{\delta g^{\sigma\sigma}} = 0\,. \tag{3.4}$$

In the light-cone gauge these constrains become

$$\begin{aligned} \partial_\sigma x^- &= \frac{1}{\alpha' p^+}\partial_\tau x^I \partial_\sigma x^I\ , \\ \partial_\tau x^- &= \frac{1}{2\,\alpha' p^+}(\partial_\tau x^I\partial_\tau x^I + \partial_\sigma x^I\partial_\sigma x^I - \lambda x_I^2)\ . \end{aligned} \tag{3.5}$$

Without loss of generality, we set $c = \frac{1}{\alpha' p^+}$ in the following. Then the bosonic action in the light-cone gauge takes the form

$$S_B = \frac{1}{4\pi\alpha'}\int d\tau \int_0^{2\pi} d\sigma\ (\partial_\tau x^I\partial_\tau x^I - \partial_\sigma x^I\partial_\sigma x^I - \tilde{f}^2 e^{-2\tau} x_I^2)\ , \tag{3.6}$$

where for simplicity we have defined $\tilde{f} \equiv \alpha' p^+ f_0$ which is dimensionless.

Obviously, in the case with $f = f_0 e^{-\phi}$, the action is exactly the same as that of the maximally supersymmetric plane-wave. Therefore the equations of motion and the canonical quantization are also the same. We will not repeat it here. However one should keep in mind that the dilaton is linear null rather than a constant and the string coupling could be very strong and the perturbative string theory is ill-defined in the strong coupling region, just like the case studied in [10]. In the remaining part of this section, we will focus on the case with constant RR field strength.

3.1 Equations of motion and modes expansion

It is easy to get the equations of motions from (3.6)

$$(\partial_\tau^2 - \partial_\sigma^2 + \tilde{f}^2 e^{-2\tau})\, x^I = 0\,. \tag{3.7}$$

Expanding in Fourier modes in σ , we get an infinite collection of oscillators with time-dependent frequencies. The general solution is given by

$$x^I(\tau,\sigma) = x_0^I(\tau) + i\sqrt{\frac{\alpha'}{2}} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \left[T_n^I(\tau)\, e^{in\sigma} - T_{-n}^I(\tau)\, e^{-in\sigma} \right], \tag{3.8}$$

with

$$\begin{aligned} x_0^I(\tau) &= J_0(\tilde{f}e^{-\tau})\, \tilde{x}^I - \frac{\pi}{2}\, \alpha'\, Y_0(\tilde{f}e^{-\tau})\, \tilde{p}^I\,, \\ T_n^I(\tau) &= Z_n(\tau)\, \alpha_n^I - Z_{-n}(\tau)\, \tilde{\alpha}_{-n}^I\,, \\ Z_n(\tau) &= \left(\frac{\tilde{f}}{2}\right)^{-in} \Gamma(1+in)\; J_{in}(\tilde{f}e^{-\tau})\,. \end{aligned} \tag{3.9}$$

Notice that $Z_{-n}(\tau) = Z_n^*(\tau)$ and $T_{-n}^I(\tau) = T_n^{I*}(\tau)$ due to the facts $\Gamma^*(1+in) = \Gamma(1-in)$ and $J_{in}^*(\tilde{f}e^{-\tau}) = J_{-in}(\tilde{f}e^{-\tau})$.

To consider the asymptotic behavior, we need use the asymptotic expansion of the Bessel functions as $z \to 0$ in (2.49). We can see that as $\tau \to +\infty$,

$$\begin{aligned} x_0^I(\tau) &\sim \tilde{x}^I + \alpha'(\tau - \ln\frac{\tilde{f}}{2})\, \tilde{p}^I\,, \\ Z_n(\tau) &\sim e^{-in\tau}\,. \end{aligned} \tag{3.10}$$

We come to the flat-space theory just as expected. Note that in our case the asymptotic flatness behavior is also shared by the zero modes, which is different from the paper [25]. Requiring that x^I are real functions implies that

$$(\alpha_n^I)^\dagger = \alpha_{-n}^I\ ,\ (\tilde{\alpha}_n^I)^\dagger = \tilde{\alpha}_{-n}^I\,. \tag{3.11}$$

The canonical momenta Π^I and the total momentum carried by the string are

$$\Pi^I = \frac{1}{2\pi\alpha'}\,\partial_\tau x^I \ , \quad p_0^I = \int_0^{2\pi} d\sigma\,\Pi^I = \frac{1}{\alpha'}\,\partial_\tau x_0^I\,. \tag{3.12}$$

To quantize the theory, we need to impose the canonical commutation relations

$$[\,x^I(\tau,\sigma),\, \Pi^J(\tau,\sigma')\,] = i\,\delta^{IJ}\delta(\sigma-\sigma')\,, \tag{3.13}$$

with the other commutators vanishing. These commutators are ensured by requiring the following commutators for the modes

$$[\tilde{x}^I,\, \tilde{p}^J] = i\,\delta^{IJ} \ , \quad [\alpha_n^I,\, \alpha_m^{J\dagger}] = \delta^{IJ}\delta_{nm} \ , \quad [\tilde{\alpha}_n^I,\, \tilde{\alpha}_m^{J\dagger}] = \delta^{IJ}\delta_{nm} \ . \tag{3.14}$$

¿From this we can also get

$$[\,x_0^I(\tau),\, p_0^J(\tau)\,] = i\,\delta^{IJ}\,. \tag{3.15}$$

Note that in these calculations we have used the formulae

$$\begin{aligned} \Gamma(1+in)\,\Gamma(1-in) &= \frac{n\pi}{\sinh n\pi} \ , \\ J_\nu(z)J'_{-\nu}(z) - J_{-\nu}(z)J'_\nu(z) &= -\frac{2\sin\nu\pi}{\pi z} \ . \end{aligned} \tag{3.16}$$

3.2 Light-cone Hamiltonian

According to (3.6) , the bosonic part of the light-cone Hamiltonian of our model is

$$H_B = \frac{1}{4\pi\alpha'^2 p^+}\int_0^{2\pi} d\sigma(\partial_\tau x^I\partial_\tau x^I + \partial_\sigma x^I\partial_\sigma x^I + \tilde{f}^2 e^{-2\,\tau}x_I^2) \ . \tag{3.17}$$

Inserting the mode expansion of x^I in terms of creation and annihilation operators, we obtain

$$\begin{aligned} H_B \;&=\; H_{B0}(\tau) + \frac{1}{2\alpha' p^+}\sum_{n=1}^{\infty}\big[\,\Omega_n^B(\tau)\,(\,\alpha_n^{I\dagger}\alpha_n^I + \tilde{\alpha}_n^{I\dagger}\tilde{\alpha}_n^I + 1) \\ &\qquad - C_n^B(\tau)\,\alpha_n^I\tilde{\alpha}_n^I - C_n^{B*}(\tau)\,\tilde{\alpha}_n^{I\dagger}\alpha_n^{I\dagger}\,\big] \ , \end{aligned} \tag{3.18}$$

with

$$\begin{aligned} H_{B0}(\tau) \;&=\; \frac{1}{2p^+}\left[(p_0^I)^2 + \tilde{f}^2 e^{-2\tau}\left(\frac{x_0^I}{\alpha'}\right)^2\right] \ , \\ \Omega_n^B(\tau) \;&=\; \frac{1}{n}|\partial_\tau Z_n|^2 + n\left(1 + \frac{\tilde{f}^2 e^{-2\tau}}{n^2}\right)|Z_n|^2 \ , \\ C_n^B(\tau) \;&=\; \frac{1}{n}(\partial_\tau Z_n)^2 + n\left(1 + \frac{\tilde{f}^2 e^{-2\tau}}{n^2}\right)(Z_n)^2 \ . \end{aligned} \tag{3.19}$$

Now let's see how the functions $\Omega^B_n(\tau)$ and $C^B_n(\tau)$ behave at infinite τ . Taking into account the asymptotic expansion of Bessel functions (2.49) , we obtain that as $\tau \to +\infty$,

$$\Omega^B_n(\tau) \sim 2n + \mathcal{O}(e^{-\tau}) \ , \ C^B_n(\tau) \sim 0 + \mathcal{O}(e^{-\tau}) \ , \tag{3.20}$$

where $C^B_n(\tau)$ is completely suppressed. We get the result just as in the flat space. On the other hand, when τ goes to $-\infty$, we come to the strongly coupled region with

$$\begin{aligned} \Omega^B_n(\tau) &\sim \frac{2\cosh n\pi}{\sinh n\pi}\, \tilde{f} e^{-\tau} , \\ C^B_n(\tau) &\sim 2\left(\frac{\tilde{f}}{2}\right)^{-2in} \frac{\Gamma^2(1+in)}{\pi}\, \tilde{f} e^{-\tau} , \end{aligned} \tag{3.21}$$

where we have used (2.47). Now $\Omega^B_n(\tau)$ and $C^B_n(\tau)$ are of the same order.

It is evident that the Hamiltonian (3.18) is non-diagonal. The treatment of the zero-mode part is the standard way used in point-particle quantization,see [25]. The non-zero mode part has non-diagonal terms proportional to $C^B_n(\tau)$ and $C^{B*}_n(\tau)$. The evolution of generic states made out of α^I_n , $\tilde{\alpha}^I_n$ is thus non-trivial. In the following we will find a new set of modes to make the Hamiltonian diagonal. Now let us introduce a new set of time-dependent string modes A^I_n , $\tilde{A}^I_n$ which are defined by

$$\begin{aligned} \frac{i}{\sqrt{|n|}}\left[Z_n \alpha^I_n - Z^*_n \tilde{\alpha}^I_{-n}\right] &= \frac{i}{\sqrt{|\,\omega_n|}}\left[e^{-i\omega_n\tau} A^I_n(\tau) - e^{i\omega_n\tau}\tilde{A}^I_{-n}(\tau)\right] \ , \\ \frac{i}{\sqrt{|n|}}\left[\partial_\tau Z_n\, \alpha^I_n - \partial_\tau Z^*_n\, \tilde{\alpha}^I_{-n}\right] &= \sqrt{|\,\omega_n|}\left[e^{-i\omega_n\tau} A^I_n(\tau) + e^{i\omega_n\tau}\tilde{A}^I_{-n}(\tau)\right] \ , \end{aligned} \tag{3.22}$$

where

$$\omega_n = \sqrt{n^2 + \tilde{f}^2 e^{-2\tau}} \ , \ n>0 \quad ; \quad \omega_{-n} = -\sqrt{n^2 + \tilde{f}^2 e^{-2\tau}} \ , \ n<0 \, , \tag{3.23}$$

with similar relations $A^\dagger_n = A_{-n}$ and $\tilde{A}^\dagger_n = \tilde{A}_{-n}$. It follows then

$$\begin{aligned} A^I_n(\tau) &= \alpha^I_n\, f_n(\tau) + \tilde{\alpha}^I_{-n}\, g^*_n(\tau) \ , \quad A^{I\dagger}_n(\tau) = \alpha^I_{-n}\, f^*_n(\tau) + \tilde{\alpha}^I_n\, g_n(\tau) \ , \\ \tilde{A}^I_n(\tau) &= \alpha^I_{-n}\, g^*_n(\tau) + \tilde{\alpha}^I_n\, f_n(\tau) \ , \quad \tilde{A}^{I\dagger}_n(\tau) = \alpha^I_n\, g_n(\tau) + \tilde{\alpha}^I_{-n}\, f^*_n(\tau) \ , \end{aligned} \tag{3.24}$$

where

$$\begin{aligned} f_n(\tau) &= \frac{1}{2}\sqrt{\frac{\omega_n}{n}}\; e^{i\omega_n\tau}\left[Z_n + \frac{i}{\omega_n}\partial_\tau Z_n\right] \ , \\ g_n(\tau) &= \frac{1}{2}\sqrt{\frac{\omega_n}{n}}\; e^{-i\omega_n\tau}\left[-Z_n + \frac{i}{\omega_n}\partial_\tau Z_n\right] \ , \end{aligned} \tag{3.25}$$

After some calculations using the commutation relations (3.14) and the properties (3.16) of the Bessel functions, we obtain the non-vanishing commutators:

$$[A_n^I \ , \ A_m^{J\dagger}] = \delta_{nm}\,\delta^{IJ} \ , \quad [\tilde{A}_n^I \ , \ \tilde{A}_m^{J\dagger}] = \delta_{nm}\,\delta^{IJ} \ . \tag{3.26}$$

In terms of these new operators, the mode expansion of $x^I(\tau,\,\sigma)$ takes the form

$$\begin{aligned} x^I(\tau,\,\sigma) = x_0^I(\tau) + i\sqrt{\frac{\alpha'}{2}}\sum_{n=1}^{\infty}\frac{1}{\sqrt{\omega_n}}\Big[& e^{-i\omega_n\tau}\left(A_n^I(\tau)\,e^{in\sigma} + \tilde{A}_n^I(\tau)\,e^{-in\sigma}\right) \\ & - e^{i\omega_n\tau}\left(A_{-n}^I(\tau)\,e^{-in\sigma} + \tilde{A}_{-n}^I(\tau)\,e^{in\sigma}\right)\Big] \ , \end{aligned} \tag{3.27}$$

and the bosonic Hamiltonian is

$$H_B = H_{B0}(\tau) + \frac{1}{\alpha' p^+}\sum_{n=1}^{\infty}\omega_n(\tau)\left[A_{-n}^I(\tau)A_n^I(\tau) + \tilde{A}_{-n}^I(\tau)\tilde{A}_n^I(\tau) + 1\right], \tag{3.28}$$

which is diagonal. The bosonic Hamiltonian reminisces the 2-d field theory of free scalars with time-dependent masses.

One may formally define the time-dependent vacuum by

$$A_n^I|0,\tau\rangle = 0, \qquad \tilde{A}_n^I|0,\tau\rangle = 0, \qquad n > 0\,, \tag{3.29}$$

and the excitation spectrum can be obtained straightforwardly. The spectrum is also time-dependent.

3.3 Classical evolution of a rotating string

To shed some light on the dynamics of the strings in our background, let us consider the behavior of the classical solutions. As an simple example, we focus on the rotating string.

First recall that in flat space, a rigid string rotating in a plane is described by a state on the leading Regge trajectory with maximal momentum for a given energy. In the light-cone gauge, the corresponding solution is

$$\begin{aligned} x^+ &= \alpha' p^+ \tau \ , \quad x^- = \alpha' p^- \tau \ , \quad L^2 = 2\,\alpha' p^+ p^- \ , \\ \mathrm{x} &= x^1 + i x^2 = L\,e^{-i\tau}\cos\sigma \ , \end{aligned} \tag{3.30}$$

where x^1 and x^2 are Cartesian coordinates of the transverse 2-plane. Here we have the standard leading Regge trajectory relation

$$-\,2\,\alpha' p^+ p^- = \alpha'(E - p_y^2) = 2\,J \ , \tag{3.31}$$

with p_y representing the momentum in the transverse directions and J standing for the corresponding angular momentum.

Now let us turn to our time-dependent background (1.1). In the case with constant λ, the analogue of the rotating string solution is given by

$$\begin{aligned} x^+ &= \alpha' p^+ \tau \ , \\ \mathrm{x} &= x^1 + i x^2 \ = \ L\, e^{-i\omega\tau} \cos\sigma \ , \end{aligned} \tag{3.32}$$

with $\omega = \sqrt{1 + (\alpha' p^+ f_0^2)}$ and x^- determined by the constraints (3.4). After some calculations, we get the light-cone Hamiltonian

$$H = -p_+ = p^- = \frac{L^2}{4\,\alpha'^2 p^+}\,\omega^2 \ , \tag{3.33}$$

and the angular momentum associated with rotations in the transverse space

$$J = \frac{1}{2\pi\alpha'} \int_0^{2\pi} d\sigma (x^1 \dot{x}^2 - \dot{x}^1 x^2) = -\frac{L^2}{2\alpha'}\,\omega \ . \tag{3.34}$$

Combining (3.33) and (3.34), we find the direct analogue of the flat-space Regge trajectory relation:

$$-\,2\,\alpha' p^+ p^- = \omega \cdot 2J\,, \tag{3.35}$$

with the "effective tension" $T = \frac{\omega}{2\pi\alpha'}$ which is still time-independent. Just as expected, this result is similar to the maximally supersymmetric case.

In the case with constant RR field strength, the analogue of the rotating string solution takes the form

$$\begin{aligned} x^+ &= \alpha' p^+ \tau \ , \\ \mathrm{x} &= x^1 + i x^2 \ = \ L\, Z_1(\tau) \cos\sigma \ , \end{aligned} \tag{3.36}$$

and x^- is again determined by the constraints (3.4) and thus evolves with time in a non-trivial way. $Z_1(\tau)$ is defined in (3.9). It is easy to check that as $\tau \to +\infty$, we recover the flat space result (3.30). In general, it describes a rotating string with effective length $L_{eff} = L\,Z_1(\tau)$ which oscillates with time and rapidly shrinks to zero as $\tau \to -\infty$.

Inserting (3.36) into the Hamiltonian (3.6) , we can get the light-cone energy

$$H = -p_+ = p^- = \frac{L^2}{4\,\alpha'^2 p^+}\Omega_1^B(\tau) \ . \tag{3.37}$$

The angular momentum associated with rotations in the transverse space is

$$J = -\frac{L^2}{2\,\alpha'} \ . \tag{3.38}$$

Combining (3.33) and (3.38) , it is interesting to note that we find

$$- 2\,\alpha' p^+ p^- = \Omega_1^B(\tau)\, J \ , \tag{3.39}$$

which is analogous to the standard leading Regge trajectory relation (3.31) but with an "effective tension" function $T = \frac{1}{2\pi\alpha'}\cdot\frac{1}{2}\Omega_1^B(\tau)$. At $\tau \to +\infty$, we have $\Omega_1^B \to 2$. Thus we reach the flat space limit. As we go back in time, the energy of this state rapidly grows until it diverges as $\tau \to -\infty$.

4 Fermionic sector

For a general plane-wave background the light-cone gauge Green-Schwarz action comes from [22, 27, 28, 29]

$$S_F = -\frac{i}{2\pi\alpha'}\int d^2\sigma(\sqrt{-g}g^{ab}\delta_{AB} - \epsilon^{ab}\sigma_{3AB})\,\partial_a x^\mu\,\bar\theta^A\Gamma_\mu(\hat D_b\theta)^B\,. \tag{4.1}$$

Here $\sigma_3 = \mathrm{diag}(1,-1)$ and $\hat D_b = \partial_b + \Omega_\nu\,\partial_b x^\nu$ is the pull-back of the generalized covariant derivative $\hat{\mathcal{D}}_\nu$ (2.30) to the world-sheet with Ω_ν defined in (2.37) in our background. In (4.1) two fermionic coordinates $\theta^{A=1,2}$ are 10d Majorana-Weyl spinors. We choose the representation of Γ-matrices such that $\Gamma^0 = C$, with C being the 10d charge conjugation, so the components of θ^A are all real. For more about our convention see Appendix A.

To fix the world-sheet gauge symmetries, in addition to the bosonic light-cone gauge conditions $\sqrt{-g}g^{ab} = \eta^{ab} = \mathrm{diag}(-1,1)$ and $x^+ = \alpha' p^+\tau$, we need to impose

$$\Gamma^+\theta^A = 0\,, \tag{4.2}$$

since the fermionic action (4.1) has an additional local κ-symmetry. These three conditions completely fix the world-sheet gauge symmetries of the fermionic action (4.1). Note that due to the gauge condition (4.2) only the second term in Ω_+ has non-vanishing contributions. After some calculations, the fermionic action in the light-cone gauge can be written as

$$S_F = \frac{i\,p^+}{\sqrt{2}\,\pi}\int d\tau\int_0^{2\pi} d\sigma\,(\theta^{1T}\partial_\tau\theta^1 + \theta^{2T}\partial_\tau\theta^2 + \theta^{1T}\partial_\sigma\theta^1 - \theta^{2T}\partial_\sigma\theta^2 + 2\tilde f e^{-\tau}\theta^{1T}\Pi\theta^2)\,. \tag{4.3}$$

Similar to the bosonic case, when $f = f_0 e^{-\phi}$, the action reduces to the one in the usual plane-wave metric. The quantization procedure follows straightforwardly. In the following we focus on the case with constant RR field strength.

4.1 Equations of motion and modes expansion

¿From the light-cone action (4.3), it is easy to get the equation of motion of the fermionic sector

$$\begin{aligned} (\,\partial_\tau + \partial_\sigma)\,\theta^1 + \tilde{f} e^{-\tau}\Pi\theta^2 = 0\ , \\ (\,\partial_\tau - \partial_\sigma)\,\theta^2 - \tilde{f} e^{-\tau}\Pi\theta^1 = 0\,. \end{aligned} \tag{4.4}$$

This is a system of two coupled first order equations, from which we can obtain two decoupled second order equations as

$$\begin{aligned} (\partial_\tau^2 - \partial_\sigma^2)\,\theta^1 + (\partial_\tau + \partial_\sigma)\,\theta^1 + \tilde{f}^2 e^{-2\tau}\theta^1 = 0\ , \\ (\partial_\tau^2 - \partial_\sigma^2)\,\theta^2 + (\partial_\tau - \partial_\sigma)\,\theta^2 + \tilde{f}^2 e^{-2\tau}\theta^2 = 0\ . \end{aligned} \tag{4.5}$$

Note that the time-dependent dilaton and the nonzero RR field strength play an important role. That is, in these two decoupled second order equations, first order differential terms appear, which do not exist in neither the maximally supersymmetric plane-wave background [23, 21, 22] due to its time independence, nor in the time-dependent plane-wave model studied in [25] because of the absence of the RR fluxes. The solutions of (4.4) are[1]

$$\begin{aligned} \theta^1(\tau,\sigma) &= \theta^1_0(\tau) + \sum_{n=1}^{\infty}\left[\,\theta^1_n(\tau)\,e^{in\sigma} + \theta^1_{-n}(\tau)\,e^{-in\sigma}\right]\ , \\ \theta^2(\tau,\sigma) &= \theta^2_0(\tau) + \sum_{n=1}^{\infty}\left[\,\theta^2_n(\tau)\,e^{in\sigma} + \theta^2_{-n}(\tau)\,e^{-in\sigma}\right]\ , \end{aligned} \tag{4.6}$$

with

$$\begin{aligned} \theta^1_0(\tau) = \frac{1}{\sqrt{4\pi\xi}}\,(\,\beta_0\cos u + \Pi\tilde{\beta}_0\sin u)\,, \\ \theta^2_0(\tau) = \frac{1}{\sqrt{4\pi\xi}}\,(\,\tilde{\beta}_0\cos u - \Pi\beta_0\sin u)\,, \end{aligned} \tag{4.7}$$

and

$$\theta^1_n(\tau) = \frac{1}{\sqrt{4\pi\xi}}\left(\beta_n W_n(\tau) + \Pi\tilde{\beta}_{-n}\,\tilde{W}^*_n(\tau)\right)\ , \tag{4.8}$$

$$\theta^2_n(\tau) = \frac{1}{\sqrt{4\pi\xi}}\left(\tilde{\beta}_{-n}\,W^*_n(\tau) - \Pi\beta_n\tilde{W}_n(\tau)\right)\ , \tag{4.9}$$

[1]Actually θ^A and $\beta_n\,,\,\tilde{\beta}_n$ etc. are all 10d spinors. If we write down the spinor indices explicitly then they should be θ^A_α and $\beta_{n\alpha}\,,\,\tilde{\beta}_{n\alpha}$ etc. with $\alpha = 1,2,\ldots,32$. Something like $\beta^\dagger_n$ and $\tilde{\beta}^\dagger_n$ should be understood as $\beta^\dagger_{n\alpha}$ and $\tilde{\beta}^\dagger_{n\alpha}$.

where for simplicity we have defined $u = \tilde{f} e^{-\tau}$, $\xi = \frac{p^+}{\sqrt{2}\pi}$ and

$$W_n(\tau) \;=\; \left(\frac{\tilde{f}}{2}\right)^{-in} \Gamma(\frac{1}{2} + in) \sqrt{\frac{u}{2}}\, J_{-\frac{1}{2}+in}(u)\,, \tag{4.10}$$

$$\tilde{W}_n(\tau) \;=\; \left(\frac{\tilde{f}}{2}\right)^{-in} \Gamma(\frac{1}{2} + in) \sqrt{\frac{u}{2}}\, J_{\frac{1}{2}+in}(u)\,. \tag{4.11}$$

The requirement that $\theta^{1,2}$ are real implies

$$\beta_{-n} = \beta_n^\dagger\ , \quad \tilde{\beta}_{-n} = \tilde{\beta}_n^\dagger\ , \quad n = 0, \pm 1, \pm 2, \ldots \tag{4.12}$$

According to the action (4.3), the canonical momentum $\mathcal{P}_\alpha^A$ conjugate to the fermionic coordinates θ_α^A are defined as (we have written down the spinor indices explicitly)

$$\mathcal{P}_\alpha^A = \frac{i\, p^+}{\sqrt{2}\,\pi}\, \theta_\alpha^A\,, \quad A = 1, 2 \quad \text{and} \quad \alpha, \beta = 1, 2, \ldots, 32\,. \tag{4.13}$$

To quantize the theory, we impose the standard anticommutation relations

$$\{\theta_{n\alpha}^A(\tau,\sigma)\,, \theta_{m\beta}^B(\tau,\sigma')\} = \frac{1}{2\,\xi}\, \delta^{AB}\, \delta_{n+m,0}\, \delta_{\alpha\beta}\, \delta(\sigma - \sigma'), \tag{4.14}$$

with the other anticommutators vanishing. These anticommutators are ensured by the following relations of the fermionic creation and annihilation operators

$$\{\beta_{n\alpha}, \beta_{m\beta}\} = \{\tilde{\beta}_{n\alpha}, \tilde{\beta}_{m\beta}\} = \delta_{\alpha\beta}\, \delta_{n+m,0}\,, \quad n, m = 0, \pm 1, \pm 2, \ldots \tag{4.15}$$

with the other anticommutators all becoming zero. Here we have used the formulae

$$\Gamma(\frac{1}{2} + in)\, \Gamma(\frac{1}{2} - in) = \frac{\pi}{\cosh n\pi}\ , \tag{4.16}$$

$$J_{-\frac{1}{2}+in}(z) J_{-\frac{1}{2}-in}(z) + J_{\frac{1}{2}+in}(z) J_{\frac{1}{2}-in}(z) = \frac{2\cosh n\pi}{\pi z}\ . \tag{4.17}$$

Now let's see whether we can recover the flat-space result when τ goes to positive infinity. Using the asymptotic expansion (2.49) of the Bessel functions , we can see that as $\tau \to +\infty$,

$$\theta_0^1(\tau) \sim \frac{1}{\sqrt{4\pi\xi}}\ \beta_0\ \ ,\ \ \theta_0^2(\tau) \sim \frac{1}{\sqrt{4\pi\xi}}\ \tilde{\beta}_0\ . \tag{4.18}$$

Therefore, just like the bosonic case, the asymptotic "flatness" behavior is also shared by the zero modes. For the oscillating modes,

$$\begin{aligned} W_n(\tau) &\sim e^{-in\tau}\ , \\ \tilde{W}_n(\tau) &\sim 0\ , \end{aligned} \tag{4.19}$$

where the second term vanishes due to the factor $e^{-\tau}$. So, for $n \neq 0$, we get the standard left and right moving plane wave. But it seems that we have lost half degrees of freedom. However this is not the case. Note that at finite τ, the constraint of the coupled 1st order equations (4.4) makes θ^1 and θ^2 not independent. The degrees of freedom are presented by β and $\tilde{\beta}$. When τ goes to positive infinity, θ^1 and θ^2 decouple. Therefore θ^1 and θ^2 both contribute to the total degrees of freedom which are still denoted by β and $\tilde{\beta}$. So we are happy to see that we have not lost any degree of freedom when we go to the flat space limit. Next we will show that the total number of degree of freedom in the fermionic sector is equal to that of the bosonic case. Recall that θ^A are ten dimensional Majorana-Weyl fermions with 16 independent real components. The constraint from fixing κ-symmetry by $\Gamma^+\theta = 0$ leaves us with each θ having 8 independent components. So the total degree of freedom is 16, just as in the bosonic case (eight x^I's with left and right moving modes).

4.2 Light-cone Hamiltonian

According to (4.3), the fermionic light-cone Hamiltonian of our model is given by

$$\begin{aligned} H_F &= -\frac{i}{\sqrt{2}\,\pi\alpha'}\int_0^{2\pi} d\sigma \left(\theta^{1T}\partial_\sigma\theta^1 - \theta^{2T}\partial_\sigma\theta^2 + 2\tilde{f}e^{-\tau}\theta^{1T}\Pi\theta^2\right) \\ &= \frac{i}{\sqrt{2}\,\pi\alpha'}\int_0^{2\pi} d\sigma \left(\theta^{1T}\partial_\tau\theta^1 + \theta^{2T}\partial_\tau\theta^2\right) \ , \end{aligned} \tag{4.20}$$

In the second line we have used the equation of motion (4.4) to simplify the expression. Inserting the mode expansion of θ^A, we rewrite the Hamiltonian in terms of creation and annihilation operators as

$$\begin{aligned} H_F = H_{F0}(\tau) + \frac{1}{2\alpha' p^+}\sum_{n=1}^{\infty}\Big[\,\Omega^F_n(\tau)\,(\,\beta^\dagger_n\beta_n + \tilde{\beta}^\dagger_n\tilde{\beta}_n - 1) \\ -C^F_n(\tau)\,\beta_n\Pi\tilde{\beta}_n - C^{F*}_n(\tau)\,\tilde{\beta}^\dagger_n\Pi\beta^\dagger_n\Big] \ , \end{aligned} \tag{4.21}$$

with

$$H_{F0}(\tau) = -\frac{2\,i}{\alpha' p^+}\,\tilde{f}\,e^{-\tau}\beta_0\Pi\tilde{\beta}_0 \ , \tag{4.22}$$

$$\Omega^F_n(\tau) = -2\,i\left[W_n\partial_\tau W^*_n + \tilde{W}_n\partial_\tau\tilde{W}^*_n\right] \ , \tag{4.23}$$

$$C^F_n(\tau) = -2\,i\left[W_n\partial_\tau\tilde{W}_n - \tilde{W}_n\partial_\tau W_n\right] \ . \tag{4.24}$$

Here although the function Ω^F_n is formally complex, we can prove that it is not only real but also positive definitely (see Appendix B), just as being expected. Now let's see the

behavior of functions $\Omega_n^F(\tau)$ and $C_n^F(\tau)$ at infinite τ. Using the asymptotic expansion of Bessel functions (2.49), we obtain as $\tau \to +\infty$,

$$\Omega_n^F(\tau) \sim 2n \quad , \quad C_n^F(\tau) \sim 0 \, . \tag{4.25}$$

Therefore we recover the flat space result. Considering the asymptotic expansion of Bessel functions (2.47), we get as $\tau \to -\infty$,

$$\begin{aligned} \Omega_n^F(\tau) &\sim \frac{2\sinh n\pi}{\cosh n\pi}\tilde{f}\, e^{-\tau} \, , \\ C_n^F(\tau) &\sim 2\, i \left(\frac{\tilde{f}}{2}\right)^{-2in} \frac{\Gamma^2(\frac{1}{2}+in)}{\pi}\tilde{f}\, e^{-\tau} \, . \end{aligned} \tag{4.26}$$

$\Omega_n^F(\tau)$ and $C_n^F(\tau)$ are of the same order in the strongly coupled region. In the following we will make the Hamiltonian diagonal.

Comparing with the diagonalization of the bosonic Hamiltonian, the diagonalization of the fermionic Hamiltonian is more tricky. We define a group of new time-dependent creation and annihilation operators, $B_n(\tau)$, $\tilde{B}_n(\tau)$ and their conjugate, by the following Bogolioubov-type transformation

$$B_n = \cos\varphi_n\ \beta_n - ie^{i\psi_n}\sin\varphi_n\ \Pi\tilde{\beta}_n^\dagger \, , \qquad B_n^\dagger = \cos\varphi_n\ \beta_n^\dagger + ie^{-i\psi_n}\sin\varphi_n\ \Pi\tilde{\beta}_n \, , \tag{4.27}$$
$$\tilde{B}_n = \cos\varphi_n\ \tilde{\beta}_n + ie^{i\psi_n}\sin\varphi_n\ \Pi\beta_n^\dagger \, , \qquad \tilde{B}_n^\dagger = \cos\varphi_n\ \tilde{\beta}_n^\dagger - ie^{-i\psi_n}\sin\varphi_n\ \Pi\beta_n \, . \tag{4.28}$$

Here φ_n and ψ_n are real functions of τ to be determined by the requirement that the fermionic Hamiltonian is diagonal. In (4.27) and (4.28) they are restricted to $n \geq 1$. For negative n we define that $B_{-n} = B_n^\dagger$ and $\tilde{B}_{-n} = \tilde{B}_n^\dagger$, which implies that $\varphi_{-n} = -\varphi_n$ and $\psi_{-n} = -\psi_n$. The operators $B_n, \tilde{B}_n$ defined in this way automatically satisfy the standard anticommutative relations[2]

$$\{B_{n\alpha}, B_{m\beta}^\dagger\} = \{\tilde{B}_{n\alpha}, \tilde{B}_{m\beta}^\dagger\} = \delta_{nm}\,\delta_{\alpha\beta} \, , \tag{4.29}$$

with the other anticommutators vanishing due to $\Pi^2 = 1$ and $\cos^2\varphi + \sin^2\varphi = 1$.

In terms of these new time-dependent creation and annihilation operators the fermionic Hamiltonian (4.3) can be written as

$$\begin{aligned} H_F \;=\; & H_{F0}(\tau) + \frac{1}{\alpha' p^+}\sum_{n=1}^{\infty}\Big[\tilde{\omega}_n(\tau)\Big(B_n^\dagger(\tau)B_n(\tau) + \tilde{B}_n^\dagger(\tau)\tilde{B}_n(\tau) - 1\Big) \\ & + K(\tau)\, B_n(\tau)\Pi\tilde{B}_n(\tau) + K^*(\tau)\,\tilde{B}_n^\dagger(\tau)\Pi B_n^\dagger(\tau)\Big] \, . \end{aligned} \tag{4.30}$$

[2]In these anticommutators we write down the spinor indices α, β explicitly.

Here the functions $\tilde{\omega}_n\,, K_n$ are

$$\tilde{\omega}_n = \frac{1}{2}\,\Omega_n^F \cos 2\varphi_n - \frac{i}{4}\,e^{i\psi_n} C_n^F \sin 2\varphi_n + \frac{i}{4}\,e^{-i\psi_n} C_n^{F*} \sin 2\varphi_n\,, \tag{4.31}$$

$$K = \frac{i}{2}\,e^{-i\psi_n}\Omega_n^F \sin 2\varphi_n - \frac{1}{2} C_n^F \cos^2\varphi_n - \frac{1}{2}\,e^{-2i\psi_n} C_n^{F*} \sin^2\varphi_n\,. \tag{4.32}$$

with Ω_n^F and C_n^F defined in (4.23) and (4.24). To diagonalize the Hamiltonian we should require the coefficient K_n of the non-diagonal terms to vanish, i.e.

$$i\,\Omega_n^F \sin 2\varphi_n = e^{i\psi_n} C_n^F \cos^2\varphi_n + e^{-i\psi_n} C_n^{F*} \sin^2\varphi_n\,. \tag{4.33}$$

We can always choose ψ_n such that

$$e^{i\psi_n} C_n^F = i|C_n^F| \quad \text{and} \quad e^{-i\psi_n} C_n^{F*} = -i|C_n^F|\,, \tag{4.34}$$

then we get the expression of the unknown function φ_n from (4.33) as following

$$\varphi_n(\tau) = \frac{1}{2}\arctan\frac{|C_n^F(\tau)|}{\Omega_n^F(\tau)}\;. \tag{4.35}$$

Now we have determined the coefficients in the definition (4.27) (4.28) of B_n and $\tilde{B}_n$. The last step is to calculate the function $\tilde{\omega}_n$ in front of the diagonal terms in (4.30) by using (4.34) and (4.35). It reads as

$$\tilde{\omega}_n = \frac{1}{2}\left[\,\Omega_n^F \cos 2\varphi_n + |C_n^F|\sin 2\varphi_n)\,\right] = \frac{1}{2}\sqrt{(\Omega_n^F)^2 + |C_n^F|^2}\;. \tag{4.36}$$

By struggling with Bessel functions we get the remarkably nice result (The details have been included in Appendix B)

$$\tilde{\omega}_n(\tau) = \omega_n(\tau) \equiv \sqrt{n^2 + \tilde{f}^2 e^{-2\tau}}\;. \tag{4.37}$$

Then the fermionic part of the Hamiltonian can be diagonalized as

$$H_F = H_{F0}(\tau) + \frac{1}{\alpha' p^+}\sum_{n=1}^{\infty}\omega_n(\tau)\left[B_n^\dagger(\tau)B_n(\tau) + \tilde{B}_n^\dagger(\tau)\tilde{B}_n(\tau) - 1\right]. \tag{4.38}$$

Just like the bosonic case, it also looks like the Hamiltonian of a free 2-d field theory with time-dependent mass.

Taking account of the bosonic sector, the full light-cone Hamiltonian takes the form

$$\begin{aligned} H &= H_0(\tau) + \frac{1}{\alpha' p^+}\sum_{n=1}^{\infty}\omega_n(\tau)\Big[\,A_n^\dagger(\tau)A_n(\tau) + \tilde{A}_n^\dagger(\tau)\tilde{A}_n(\tau) \\ &\qquad + B_n^\dagger(\tau)B_n(\tau) + \tilde{B}_n^\dagger(\tau)\tilde{B}_n(\tau)\,\Big]\,, \end{aligned} \tag{4.39}$$

with

$$H_0(\tau) = H_{B0}(\tau) + H_{F0}(\tau) \ . \tag{4.40}$$

We can see that the zero-point energy exactly cancels between the bosons and the fermions. Just like the bosonic case, one may define the fermionic spectrum formally, which is also time-dependent. From the total Hamiltonian, it is obvious that the spectrum is symmetric between the bosonic and fermionic excitations.

In this and the last section, we have discussed the quantization of the free bosonic and fermionic strings in the plane-wave background with a linear null dilaton. In the lightcone gauge, the calculation of perturbative string amplitudes is simple formally. In the case with constant λ, the spectrum is exactly the same as the ones in maximally supersymmetric plane-wave case, and the perturbative amplitudes in the lightcone gauge is also very similar except the overall dependence on the string coupling. The dependence on the string coupling shows that the perturbative calculation is ill-defined in the strong coupling region. In the case with constant RR field strength, it is more subtle. When $\tau \to -\infty$, not only the string coupling is very strong, but also the energy of the excitations is huge. We wish the perturbative amplitude calculation is still well-defined in the weak coupling region.

5 Quantum string mode creation

Generically, in a time-dependent background, one may expect the particle or string creation occurs from our knowledge of the quantum field theory in curved spacetime. However, in a plane-fronted background, due to the existence of null Killing vector, this would not happen[30]. Nevertheless, as pointed out in [20], there does exist the string mode creation. This could be easily seen from the time-dependent Hamiltonian we have obtained above. From the point of view of two-dimensional quantum field theory, we have a time-dependent potential, which induces the transition between different modes of the string. See also [33, 34, 35, 36].

Now let us study quantum string mode creation in our background (1.1). In general, given a pp-wave background with asymptotically flat region at $\tau = +\infty$, a string starting in a certain state at $\tau = +\infty$ evolves back with time and maybe end up in a different sate. Equivalently, one may reverse the orientation of time (which is equal to change the sigh of c in $\phi = -cx^+$) and interpret this as an evolution from some excited state to the vacuum at $\tau = +\infty$. The main reason is that the string may interact with the pp-wave background to have extra internal excitations.

In the following we will consider whether an observer in the "in" vacuum $|0\rangle_\infty$ at $\tau = +\infty$ will see string mode creation. Here the vacuum $|0\rangle_\infty$ is the Fock space state which is annihilated by the operators α_n^I , $\tilde{\alpha}_n^I$ defined in (3.14) and β_n^I , $\tilde{\beta}_n^I$ defined in (4.14). We shall start with the string in $|0\rangle_\infty$ state at $\tau = +\infty$ and study how this state should evolve back to $\tau = -\infty$. In other words, we will calculate the probability of a string state $|n, \tau = -\infty >$ transiting into the vacuum state at $\tau = \infty$.

First let us see the expectation value of the "oscillator number" operator that appears in the bosonic Hamiltonian (3.28)

$$\begin{aligned} \bar{N}_n^B(\tau) &\equiv {}_\infty\langle 0|[\,A_n^{I\dagger}(\tau)A_n^I(\tau) + \tilde{A}_n^{I\dagger}(\tau)\tilde{A}_n^I(\tau)]|0\rangle_\infty \\ &= 2\,d\,g_n^*(\tau)g_n(\tau) \ , \end{aligned} \tag{5.1}$$

where we have used (3.24) and $d = 8$, the range of index I. Inserting the definition (3.25) of $g_n(\tau)$, we find

$$\bar{N}_n^B(\tau) = d\left[\frac{\Omega_n^B}{2\,\omega_n} - 1\right] \ . \tag{5.2}$$

Using the same method, we get the expectation value of the "oscillator number" operator in the fermionic Hamiltonian (4.38)

$$\begin{aligned} \bar{N}_n^F(\tau) &\equiv {}_\infty\langle 0|[\,B_n^{I\dagger}(\tau)B_n^I(\tau) + \tilde{B}_n^{I\dagger}(\tau)\tilde{B}_n^I(\tau)]|0\rangle_\infty \\ &= d\left[1 - \frac{\Omega_n^F}{2\,\omega_n}\right] \ , \end{aligned} \tag{5.3}$$

Therefore the total number of created oscillator modes is

$$\begin{aligned} \bar{N}_T(\tau) &= \sum_{n=1}^{\infty}\left[\bar{N}_n^B + \bar{N}_n^F\right] \ , \\ &= d\sum_{n=1}^{\infty}\left[\frac{\Omega_n^B - \Omega_n^F}{2\,\omega_n}\right] \ . \end{aligned} \tag{5.4}$$

As $\tau \to +\infty$,

$$\omega_n(\tau) \sim n \quad , \quad \Omega_n^B(\tau) \sim 2\,n \quad , \quad \Omega_n^F(\tau) \sim 2\,n \ , \tag{5.5}$$

we can see

$$\bar{N}_T(\tau) \sim 0 \ . \tag{5.6}$$

This is what we want since now $A_n^I(\tau)$, $\tilde{A}_n(\tau)$ are the same as α_n^I, $\tilde{\alpha}_n^I$ and $B_n(\tau)$, $\tilde{B}_n(\tau)$ are the same as β_n, $\tilde{\beta}_n$; we do not expect to see string modes creation in flat space. As $\tau \to -\infty$,

$$\bar{N}_n^B(\tau) \sim \frac{2\,d\,e^{-n\pi}}{e^{n\pi} - e^{-n\pi}} \quad , \quad \bar{N}_n^F(\tau) \sim \frac{2\,d\,e^{-n\pi}}{e^{n\pi} + e^{-n\pi}} \ , \tag{5.7}$$

the total number of created oscillator modes is

$$\begin{aligned}\bar{N}_T(\tau) &= 4d\sum_{n=1}^{\infty}\frac{1}{e^{2n\pi}-e^{-2n\pi}} \\ &\sim 0.06\ . \end{aligned}\tag{5.8}$$

Therefore there is nearly no string mode creation as $\tau \to -\infty$.

Here is an effective way to understand the above result. As pointed out in [20], the problem could be restated as a quantum mechanical problem. Let us focus on the bosonic sector for brevity. ¿From the equation of motion, we know that the $T_n(\tau)$ in (3.8) should satisfy the equation

$$\partial_\tau^2 T_n + (\,n^2 + \tilde{f}^2 e^{-2\tau})\,T_n = 0. \tag{5.9}$$

Replacing τ by x and T_n by ψ, the above equation takes a form of one-dimensional Schrodinger equation for a particle with energy n^2 in a potential $-\tilde{f}^2 e^{-2\tau}$. The problem of calculating the number of the creating modes reduces to the problem of calculating the reflective amplitude in this one-dimensional system. Here the in-coming wave is a plane-wave from the $x=\infty$ and there is a very deep well near the $x=-\infty$. Obviously the probability of the reflection is very low. This is consistent with the above calculation. Note that the large tidal force near the $\tau = -\infty$ is attractive rather than repulsive, this makes the difference. In the repulsive case, one has a large potential barrier in the corresponding quantum mechanical system, and the reflective amplitude is large so that the string mode transition is violent.

Furthermore, it is remarkable that the linear null dilaton does not affect the discussion on the string mode creation. As shown in [20], the effect of the dilaton in the equation of the string evolution is in a form of double derivatives. For the linear dilaton, it has no effect on the string evolution.

6 Concluding remarks

In this paper, we studied the perturbative string theory in a time-dependent plane-wave background with a constant RR field strength and a linear null dilaton. In the light-cone gauge, we obtain two-dimensional free massive field theories with time-dependent masses in both the bosonic and fermionic sectors. Following the standard canonical quantization, we obtained a time-dependent Hamiltonian with vanishing zero-point energy. The spectrum also is time-dependent and symmetric between bosonic and fermionic excitations. Finally we investigated the string mode creation in our background and found that it is negligible.

It is remarkable that the symmetric spectrum of the bosonic and fermionic excitations does not come from the spacetime supersymmetries. As it is well-known, even in the case with supernumerary supersymmetries, the excitation spectrum of the bosonic sector and fermionic sector is in general different. This is because that the masses of the transverse bosons in the action depends on the form of the metric and could be different from each other. On the other hand, the fermionic action depends only on the RR field strength in the lightcone gauge so that the masses of the fermionic excitations are the same among the transverse directions. Therefore the symmetric spectrum found in this paper is from the special choice of the metric in (1.1). One may consider the following background

$$\begin{aligned} ds^2 &= -2dx^+dx^- - \sum_I \lambda_I(x^+)\, x_I^2\, dx^+dx^+ + dx^I dx^I\,, \\ \phi &= \phi(x^+)\,, \qquad (F_5)_{+1234} = (F_5)_{+5678} = 2f. \end{aligned} \tag{6.1}$$

Here λ_I could be different from each other. The conformal invariance condition requires that

$$\sum_I \lambda_I = -2\phi'' + 8f^2 e^{2\phi}. \tag{6.2}$$

It is not hard to find that such backgrounds still keep sixteen supersymmetries characterized by the Killing spinor ϵ satisfying $\Gamma^+\epsilon = 0$. The string theory in these backgrounds could be solved exactly in the lightcone gauge, at least for the cases with a constant RR field strength or the constant λ_I. The straightforward calculation shows that the transverse bosons in the bosonic action, whose form is similar to (3.6), have different masses proportional to λ_I. Therefore, the bosonic excitations are different from each other. It is impossible to have symmetric spectrum between bosonic and fermionic sector any more. As a consequence, the zero-point energy cannot be cancelled exactly.

Our study is just the first step to understand string theory in such backgrounds. As we have seen, there exists the cosmological singularity at $x^+ = -\infty$, where the string coupling is divergent. This indicates that the perturbative string description breaks down and we need other degrees of freedom to describe the physics there. In the strong coupling region, the nonperturbative degrees of freedom are essential to describe the physics. In the spirit of BFSS conjecture of M-theory[38], the matrix model, which describe the dynamics of the partons, is a natural candidate. There are a few recent papers studying the matrix models in various time-dependent backgrounds[10, 11, 12, 13]. In particular, in [12, 13], the 1/2-BPS plane-wave backgrounds of the 11D supergravity have been found and the corresponding matrix models have been constructed. It would be very interesting to construct the matrix description of IIB backgrounds in this paper. This is under our

investigation. Comparing with the cases in [12, 13], one advantage of the backgrounds discussed here is that they are solvable. With the perturbative study in this paper, it would be illuminating to see how the matrix degrees of freedom at the big bang are frozen to those of the perturbative strings in the late time.

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A Notation

In this appendix we collect our convention of the 10d Gamma matrices with respect to the vierbeins, not to the coordinates. Therefore the Gamma matrices defined here all carry hatted indices $\hat{\mu}, \hat{\nu}$ etc. They satisfy the standard anticommutators

$$\{\Gamma^{\hat{\mu}}, \Gamma^{\hat{\nu}}\} = 2\,\eta^{\hat{\mu}\hat{\nu}}, \quad \hat{\mu}, \hat{\nu} = +, -, I, \quad I = 1, 2, \ldots, 8. \tag{A.1}$$

where

$$\eta^{\hat{\mu}\hat{\nu}} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & \mathbf{1}_8 \end{pmatrix}, \tag{A.2}$$

i.e. $(\Gamma^{\hat{+}})^2 = (\Gamma^{\hat{-}})^2 = 0$, $\{\Gamma^{\hat{+}}, \Gamma^{\hat{-}}\} = -2$, $\{\Gamma^{\hat{I}}, \Gamma^{\hat{J}}\} = 2\delta^{IJ}$. We also define Γ^0 and Γ^9 by $\Gamma^{\hat{\pm}} = (\Gamma^0 \pm \Gamma^9)/\sqrt{2}$. We can choose the representation of Gamma matrices as

$$\Gamma^{\hat{+}} = i\begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}, \quad \Gamma^{\hat{-}} = i\begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}, \quad \Gamma^{\hat{I}} = \begin{pmatrix} \gamma^I & 0 \\ 0 & -\gamma^I \end{pmatrix}. \tag{A.3}$$

The 10d chiral matrix Γ is

$$\Gamma = \begin{pmatrix} \gamma & 0 \\ 0 & -\gamma \end{pmatrix}. \tag{A.4}$$

Here γ^I's satisfy $\{\gamma^I, \gamma^J\} = 2\delta^{IJ}$, which is a representation of $SO(8)$ Clifford algebra. We can choose them as real and symmetric 8×8 matrices. γ is the chiral matrix of γ^I's.

In our representations we have $C = \Gamma^0$ with C being the 10d charge conjugation matrix. Therefore the components of Majorana spinors in this representation are all real. This property renders many things simple. We also define $\Gamma_{\hat{\mu}} = \eta_{\hat{\mu}\hat{\nu}}\Gamma^{\hat{\nu}}$, so

$$\Gamma_{\hat{+}} = -\Gamma^{\hat{-}}, \quad \Gamma_{\hat{-}} = -\Gamma^{\hat{+}}, \quad \Gamma_{\hat{I}} = \Gamma^{\hat{I}}. \tag{A.5}$$

The fermionic light-cone gauge condition $\Gamma^{\hat{+}}\theta^A = 0$ implies, in this representation, that the latter 16 components of the fermionic coordinates θ^A all vanish, i.e.

$$\theta^A = \begin{pmatrix} \vartheta^A \\ 0 \end{pmatrix}, \quad A = 1, 2\,. \tag{A.6}$$

Then the positive 10d chirality $\Gamma\theta^A = \theta^A$ implies that $\gamma\,\vartheta^A = \vartheta^A$. Since we can choose the representation of γ^I properly such that $\gamma = \mathrm{diag}(1, -1)$, so $\gamma\,\vartheta^A = \vartheta^A$ is followed by the consequence that only the first 8 components of ϑ^A, also θ^A, are nonzero. Therefore we end up with two $SO(8)$ Majorana-Weyl spinors both in the same chiral representation. For more aspects of this representation, see the reference [37].

B Proof of two propositions

In this section we give the proof of two claims

$$(1) \quad \Omega_n^F = -2\,i\left[W_n\partial_\tau W_n^* + \tilde{W}_n\partial_\tau\tilde{W}_n^*\right] \text{ is positive definitely.}$$

$$(2) \quad (\Omega_n^F)^2 + |C_n^F|^2 = 4n^2 + 4u^2\,, \quad u = \tilde{f}e^{-\tau}.$$

For convenience we first list some properties of the Bessel function $J_\nu(z)$. The Bessel function $J_\nu(z)$ is the solution of following the Bessel equation (about $y(z)$)

$$\frac{1}{z}\frac{d}{dz}\left(z\frac{dy}{dz}\right) + \left(1 - \frac{\nu^2}{z^2}\right)y = 0\,. \tag{B.1}$$

The Bessel function $J_\nu(z)$ has the following relations

$$zJ_{\nu-1} = \nu J_\nu + zJ'_\nu\,, \tag{B.2}$$

$$zJ_{\nu+1} = \nu J_\nu - zJ'_\nu\,. \tag{B.3}$$

¿From these two formulae it has

$$\frac{2\nu}{z}J_\nu = J_{\nu-1} + J_{\nu+1}\,, \tag{B.4}$$

$$2\,J'_\nu = J_{\nu-1} + J_{\nu+1}\,. \tag{B.5}$$

The Wronskian determinant of $J_{\pm\nu}(z)$ is

$$W[J_\nu, J_{-\nu}] \equiv J_\nu J'_{-\nu} - J'_\nu J_{-\nu} = -\frac{2\sin\nu\pi}{\pi z}\,. \tag{B.6}$$

By using of (B.2) and (B.6) we have

$$J_{-\frac{1}{2}+in}(z)\, J_{-\frac{1}{2}-in}(z) + J_{\frac{1}{2}+in}(z)\, J_{\frac{1}{2}-in}(z) = \frac{2\cosh n\pi}{\pi z}\,. \tag{B.7}$$

In general the parameter ν and the argument z are both complex numbers. For real argument $z = u \in \mathbb{R}$ we have $J^*_\nu(u) = J_{\nu^*}(u)$.

B.1 Proof of (1)

Firstly we can simplify the expression of Ω^F_n as

$$\begin{aligned} \Omega^F_n &= \frac{2i\pi u}{\cosh n\pi}\left\{\frac{\cosh n\pi}{2\pi u} + G(u)\right\} \\ &\equiv \frac{2i\pi u}{\cosh n\pi}\left\{\frac{\cosh n\pi}{2\pi u} + \frac{u}{2}\left[J_{-\frac{1}{2}+in}(u)\, J'_{-\frac{1}{2}-in}(u) + J_{\frac{1}{2}+in}(u)\, J'_{\frac{1}{2}-in}(u)\right]\right\}. \end{aligned} \tag{B.8}$$

Here we have used (B.7) and defined the function G as in the second line. The real part of G is

$$\begin{aligned} \mathrm{Re}\, G &= \frac{1}{2}\,\{G + G^*\} \\ &= \frac{u}{4}\frac{d}{du}\left\{|J_{-\frac{1}{2}+in}|^2 + |J_{\frac{1}{2}+in}|^2\right\} \\ &= -\frac{\cosh n\pi}{2\pi u}\,, \end{aligned} \tag{B.9}$$

which cancels the first term in the first line of (B.8). Then we have proved that Ω^F_n is real. Secondly we should show that the remaining part

$$\Omega^F_n = -\frac{2\,\pi\, u}{\cosh n\pi}\ \mathrm{Im}\, G \tag{B.10}$$

is positive definitely for all $u > 0$. To do so we consider of the derivative of ImG. It is not difficult to get that

$$\frac{d}{du}\,\mathrm{Im}G = \frac{n}{2u}\left\{|J_{-\frac{1}{2}+in}|^2 + |J_{\frac{1}{2}+in}|^2\right\} > 0\,, \quad \text{for } u > 0\,. \tag{B.11}$$

Here we have used the Bessel equation (B.1) to eliminate the second derivative terms. The limit of ImG as $u \to +\infty$ is $\frac{1-e^2}{2\pi e} < 0$, which can be obtained by the standard formulae of asymptotic behaviors of Bessel functions. Then ImG is negative definitely. Therefore we have prove that Ω^F_n, although is complex formally, is not only real but also positive definitely for all $u > 0$.

B.2 Proof of (2)

Here we give the details of the proof of (4.37) which states that

$$(\Omega_n^F)^2 + |C_n^F|^2 = 4n^2 + 4u^2 \,, \quad u = \tilde{f} e^{-\tau} > 0 \,. \tag{B.12}$$

Using the definitions (4.23) and (4.24) of Ω_n^F and C_n^F, it is not difficult to know that

$$(\Omega_n^F)^2 + |C_n^F|^2 = \frac{4\pi u^2}{\cosh n\pi}(A + B + C) \tag{B.13}$$

with

$$\begin{aligned} A &= \frac{1}{8u}\left\{ \left| J_{-\frac{1}{2}+in}(u) \right|^2 + \left| J_{\frac{1}{2}+in}(u) \right|^2 \right\} = \frac{\cosh n\pi}{4\pi u^2} \,, && \text{(B.14)} \\ B &= \frac{1}{4}\frac{d}{du}\left\{ \left| J_{-\frac{1}{2}+in}(u) \right|^2 + \left| J_{\frac{1}{2}+in}(u) \right|^2 \right\} = -\frac{\cosh n\pi}{2\pi u^2} \,, && \text{(B.15)} \\ C &= \frac{u}{2}\left\{ \left| J'_{-\frac{1}{2}+in}(u) \right|^2 + \left| J'_{\frac{1}{2}+in}(u) \right|^2 \right\} . && \text{(B.16)} \end{aligned}$$

For the second equalities of function A and B we have used the identity (B.7). To simplify the function C we utilize the formula (B.2) and (B.3) that

$$\begin{aligned} J'_{-\frac{1}{2}\pm in} &= \frac{-\frac{1}{2} \pm in}{u} J_{-\frac{1}{2}\pm in} - J_{\frac{1}{2}\pm in} \,, && \text{(B.17)} \\ J'_{\frac{1}{2}\pm in} &= -\frac{\frac{1}{2} \pm in}{u} J_{\frac{1}{2}\pm in} + J_{-\frac{1}{2}\pm in} \,, && \text{(B.18)} \end{aligned}$$

and the formula (B.7), then we have

$$C = \frac{1 + 4n^2 + 4u^2}{4\pi u^2} \,. \tag{B.19}$$

Put all together we have proved what we want, i.e.

$$(\Omega_n^F)^2 + |C_n^F|^2 = 4n^2 + 4u^2 \,, \quad u = \tilde{f} e^{-\tau} > 0 \,. \tag{B.20}$$

Therefore the zero-point energy really cancels between bosons and fermions.

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